ARTICLE 65

Formulae for the geometrical and yod populations of the outer and inner forms of N Trees of Life/N-tree composed of any order of polygon

by

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Abstract

Formulae are derived for the populations of geometrical elements and yods in a polygon divided any number of times into triangles. The populations of N overlapping Trees of Life, the N-tree and their inner form as N sets of (7+7) enfolded, regular polygons are calculated. The ratio of the yod and geometrical populations of a set of seven enfolded polygons is found to be 3/2, independent of the number of times they are divided. This is the tone ratio of the perfect fifth, the primary note in a musical octave. For any number n of their divisions into triangles, the corresponding ratio for the (7+7) enfolded polygons is within 0.1% of this fraction, converging towards it in the asymptotic limit as n approaches infinity. The integers 1, 2, 3 & 4 symbolised by the four rows of the Pythagorean tetractys determine various populations of yods and geometrical elements in the inner Tree of Life. ADONAI, the Divine Name assigned to Malkuth, is found to prescribe the population of yods and geometrical elements in the combined outer & inner forms of N Trees and the N-tree.

Table 1. Gematria number values of the ten Sephiroth in the four Worlds.

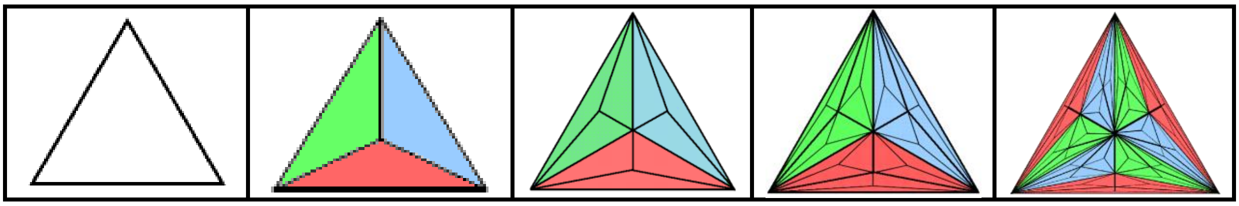
	SEPHIRAH	GODNAME	ARCHANGEL	ORDER OF ANGELS	MUNDANE CHAKRA
1	Kether (Crown) 620	EHYEH (I am) 21	Metatron (Angel of the Presence) 314	Chaioth ha Qadesh (Holy Living Creatures) 833	Rashith ha Gilgalim First Swirlings. (Primum Mobile) 636
2	Chokmah (Wisdom) 73	YAHWEH, YAH (The Lord) 26, 15	Raziel (Herald of the Deity) 248	Auphanim (Wheels) 187	Masloth (The Sphere of the Zodiac) 140
3	Binah (Understanding) 67	ELOHIM (God in multiplicity) 50	Tzaphkiel (Contemplation of God) 311	Aralim (Thrones) 282	Shabathai Rest. (Saturn) 317
	Daath (Knowledge) 474				
4	Chesed (Mercy) 72	EL (God) 31	Tzadkiel (Benevolence of God) 62	Chasmalim (Shining Ones) 428	Tzadekh Righteousness. (Jupiter) 194
5	Geburah (Severity) 216	ELOHA (The Almighty) 36	Samael (Severity of God) 131	Seraphim (Fiery Serpents) 630	Madim Vehement Strength. (Mars) 95
6	Tiphareth (Beauty) 1081	YAHWEH ELOHIM (God the Creator) 76	Michael (Like unto God) 101	Malachim (Kings) 140	Shemesh The Solar Light. (Sun) 640
7	Netzach (Victory) 148	YAHWEH SABAOTH (Lord of Hosts) 129	Haniel (Grace of God) 97	Tarshishim or Elohim 1260	Nogah Glittering Splendour. (Venus) 64
8	Hod (Glory) 15	ELOHIM SABAOTH (God of Hosts) 153	Raphael (Divine Physician) 311	Beni Elohim (Sons of God) 112	Kokab The Stellar Light. (Mercury) 48
9	Yesod (Foundation) 80	SHADDAI EL CHAI (Almighty Living God) 49, 363	Gabriel (Strong Man of God) 246	Cherubim (The Strong) 272	Levanah The Lunar Flame. (Moon) 87
10	Malkuth (Kingdom) 496	ADONAI MELEKH (The Lord and King) 65, 155	Sandalphon (Manifest Messiah) 280	Ashim (Souls of Fire) 351	Cholem Yesodoth The Breaker of the Foundations. The Elements. (Earth) 168

(All numbers in this table that the article refers to are written in ***boldface****).*

The Sephiroth exist in the four Worlds of Atziluth, Beriah, Yetzirah and Assiyah. Corresponding to them are the Godnames, Archangels, Order of Angels and Mundane Chakras (their physical manifestation, traditionally symbolised by celestial bodies). This table gives their number values obtained by the ancient practice of gematria, wherein a number is assigned to each letter of the alphabet, thereby giving to a word a number value that is the sum of the numbers of its letters.

1. Geometrical composition of N Trees/N-tree with nth-order triangles

Triangles are the basic building blocks of geometry. But in sacred geometry they can be considered not only as pure triangles (what we may call "0th-order triangles") but also as triangles divided into their three sectors (what in previous articles have been called "Type A triangles" but which will here be called "1st-order triangles"). If each sector of the Type A triangle is then divided into its three sectors, it becomes a "Type B triangle," or a "2nd-order triangle". It is composed of (3^2=9) 0th-order triangles. The next division into three sectors of each triangle in a Type B triangles generates a "Type C triangle" ("3rd-order triangle"), and so on. An nth-order triangle is the result of n successive divisions of a 0th-order triangle into 3^n 0th-order triangles:



0^{th}-order triangle	1^{st}-order triangle (Type A)	2^{nd}-order triangle (Type B)	3^{rd}-order triangle (Type C)	4^{th}-order triangle (Type D)

Table 2 shows how the numbers of corners, sides & triangles *inside* the parent triangle increase with n:

Table 2.

	n = 0	n = 1	n = 2	n = 3	n = n
Number of corners (c)	0	3	$3^0 + 3^1$	$3^0 + 3^1 + 3^2$	$3^0 + 3^1 + 3^2 + ... + 3^{n-1} = ½(3^n-1)$
Number of sides (s)	0	3^1	$3^1 + 3^2$	$3^1 + 3^2 + 3^3$	$3^1 + 3^2 + 3^3 + ... + 3^n = (3/2)(3^n-1)$
Number of 0th-order triangles (t)	3^0	3^1	3^2	3^3	3^n
Number of geometrical elements (g=c+s+t)	1	7	25	79	$3^{n+1} - 2$

Geometrical composition

Table 3 lists the numbers of corners, sides & 0th-order triangles in N Trees of Life or in the N-tree:

Table 3

	N Trees of Life	N-tree
Number of corners (C)	6N + 4	6N + 5
Number of sides (S)	16N + 6	16N + 9
Number of triangles (T)	12N + 4	12N + 7
Number of geometrical elements (G)	34N + 14	34N + **21**

The number of corners of the Tt triangles in N Trees/N-tree with T nth-order triangles ≡ C = C + Tc.
The number of sides of the Tt triangles in N Trees/N-tree ≡ S = S + Ts.
The number of 0th-order triangles in N Trees/N-tree ≡ T = Tt.
The number of geometrical elements in N Trees/N-tree ≡ G = C + S + T = C + S + T(c+s+t) = C + S + Tg.

Table 4 lists the calculated geometrical composition of N Trees/N-tree with nth-order triangles:

Table 4

	N Trees of Life	N-tree
Number of corners (C)	$2 + 2(3N+1)3^n$	$½[3 + (12N+7)3^n]$
Number of sides (S)	$-2N + 2(3N+1)3^{n+1}$	$½[-(4N+3) + (12N+7)3^{n+1}]$
Number of triangles (T)	$4(3N+1)3^n$	$(12N+7)3^n$
Number of geometrical elements (G)	$2 - 2N + 4(3N+1)3^{n+1}$	$-2N + (12N+7)3^{n+1}$

The cases N = 1 and N =10 are of special interest because 10 overlapping Trees of Life are an expanded version of the single Tree of Life. Table 5 lists their geometrical compositions for 0th- & 1st-order triangles:

Table 5

	N Trees of Life				N-tree			
	Corners	Sides	Triangles	Total	Corners	Sides	Triangles	Total
n = 0 (all triangles are tetractyses)								
N = 1	10	22	16	**48**	11	25	19	55

N = 10	**64**	166	124	354	**65**	169	127	361
n = 1 (all triangles are Type A)								
N = 1	**26**	70	**48**	144	30	82	57	169
N = 10	188	538	372	1098	192	550	381	1123

Table 6 lists the geometrical composition of N Trees/N-tree for the orders n = 0, 1, 2, & 3:

Table 6

	N Trees of Life				**N-tree**			
n	**Corners**	**Sides**	**Triangles**	**Total**	**Corners**	**Sides**	**Triangles**	**Total**
0	6N + 4	16N + 6	12N + 4	34N + 14	6N + 5	16N + 9	12N + 7	34N + **21**
1	18N + 8	52N + 18	**36**N + 12	106N + 38	18N + 12	52N + 30	**36**N + **21**	106N + 63
2	54N + 20	160N + 54	108N + **36**	322N + 110	54N + 33	160N + 93	108N + 63	322N + 189
3	162N + 56	484N + 162	324N + 108	970N + 326	162N + 96	484N + **282**	324N + 189	970N + 567

Yod composition

The number of hexagonal yods inside each of the T nth-order triangles = 2s + t.
The number of hexagonal yods lining the outer sides of these nth-order triangles = 2S.
The number of hexagonal yods in N Trees/N-tree ≡ H = 2S + T(2s+t).
The number of yods in N Trees/N-tree ≡ Y = C + 2S + T(2s+t).

Table 7 lists the yod population of N Trees/N-tree with nth-order triangles:

Table 7. Yod population of N-trees/N-tree with nth-order triangles.

	N Trees of Life	**N-tree**
Number of corners (C)	$2 + 2(3N+1)3^n$	$½[3 + (12N+7)3^n]$
Number of hexagonal yods (H)	$−4N + 16(3N+1)3^n$	$−(4N+3) + 4(12N+7)3^n$
Number of yods (Y)	$2 − 4N + 2(3N+1)3^{n+2}$	$½[−(8N+3) + (12N+7)3^{n+2}]$

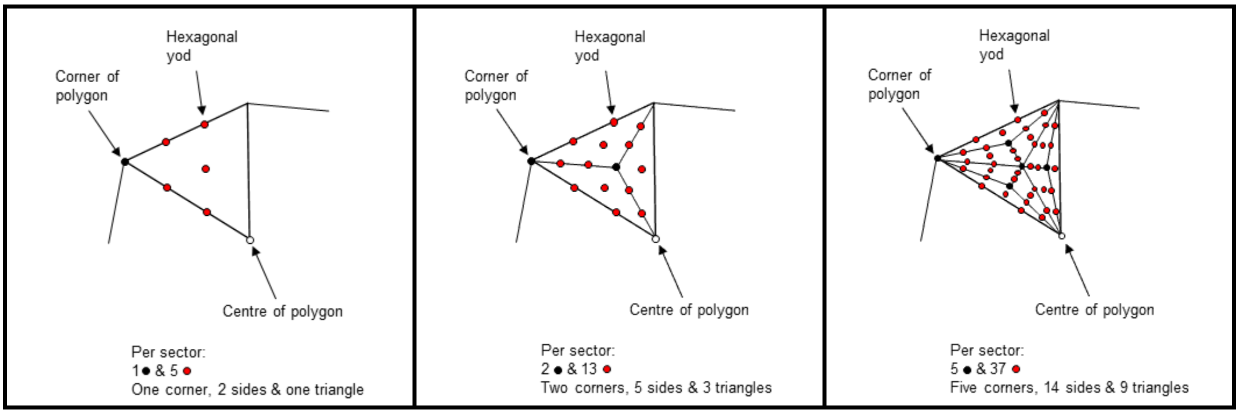
Table 8 lists the yod populations (Y) for N Trees/N-tree in the case of n = 0 (tetractys), n = 1 (Type A triangle), n = 2 (Type B triangle) & n = 3 (Type C triangle):

Table 8. Yod populations of N Trees/N-tree for n = 0, 1, 2 & 3.

n	**N Trees of Life**	**N-tree**
0	**50**N + 20	**50**N + 30
1	158N + 56	158N + 93
2	482N + 164	482N + **282**
3	1454N + 488	1454N + 849

2. The geometrical & yod compositions of polygons

Let us define a 1st-order polygon P_1 as a polygon divided into its sectors, a 2nd-order polygon P_2 as a P_1 whose sectors are each divided into three sectors, etc. P_1 is the Type A polygon, P_2 is the Type B polygon, P_3 is the Type C polygon, etc. A sector of the first three orders of polygons is shown below:



1st-order polygon 2nd-order polygon 3rd-order polygon

The number of corners per sector of the nth-order polygon P_n (n = 1, 2, 3, 4, etc.) increases with n as 1,

2, 2 + 3^1, 2 + 3^2, etc. The number of sides increases as 2, 2 + 3^1, 2 + 3^1 + 3^2, 2 + 3^1 + 3^{2+} 3^3, etc. The number of triangles increases as 1, 3^1, 3^2, 3^3, etc. Including its centre, P_n has c_n corners, s_n sides & t_n triangles, where:

$$c_n = 1 + 3^0 + 3^1 + ... + 3^{n-2} = (1+3^{n-1})/2,$$

$$s_n = 1 + 3^0 + 3^1 + + ... + 3^{n-1} = (1+3^n)/2,$$

and

$$t_n = 3^{n-1}.$$

Therefore,

$$c_n + t_n = (1+3^{n-1})/2 + 3^{n-1} = (1+3^n)/2 = s_n,$$

i.e., the number of corners & sides per sector of P_n is equal to the number of sides per sector of the triangles in each sector. This means that the *number of corners & triangles surrounding the centre of P_n is equal to the number of sides of its triangles*. This division of the geometrical elements surrounding the centre of a polygon into two *equal* sets is highly significant, having important implications that are revealed in the discussion elsewhere in this website of how sacred geometries encode the group mathematics and structure of $E_8 \times E_8$ heterotic superstrings.

The number of corners, sides & triangles per sector ≡ $g_n = c_n + s_n + t_n = 2s_n = 1 + 3^n$.

Including its centre, the number of corners in an nth-order m-gon ≡ $C_n(m) = 1 + mc_n = 1 + m(1+3^{n-1})/2$.

The number of sides ≡ $S_n(m) = ms_n = m(1+3^n)/2$.

The number of triangles ≡ $T_n(m) = mt_n = 3^{n-1}m$.

The number of geometrical elements in an nth-order m-gon ≡ $G_n(m) = C_n(m) + S_n(m) + T_n(m)$

$$= 1 + mg_n = 1 + m(1+3^n).$$

The number of hexagonal yods in an nth-order m-gon ≡ $h_n(m) = 2S_n(m) + T_n(m) = m(1+3^{n-1}+3^n)$.

Including the centre of the m-gon, the number of yods ≡ $Y_n(m) = C_n(m) + h_n(m) = 1 + m(3+3^{n+1})/2$.

For reference, Table 9 lists the formulae for the geometrical and yod populations of the nth-order m-gon:

Table 9. Geometrical & yod compositions of the nth-order m-gon.

Number of corners (including centre of m-gon)	$C_n(m) = 1 + m(1+3^{n-1})/2$
Number of sides	$S_n(m) = m(1+3^n)/2$
Number of triangles	$T_n(m) = 3^{n-1}m$
Total number of geometrical elements	$G_n(m) = 1 + m(1+3^n)$
Number of hexagonal yods	$h_n(m) = m(1+3^{n-1}+3^n)$
Total number of yods	$Y_n(m) = 1 + 3m(1+3^n)/2$

("1" denotes the centre of the nth-order m-gon).

Seven separate polygons

The seven types of regular polygons present in the inner form of the Tree of Life have 48 corners. The number of geometrical elements in the seven separate nth-order polygons = $\sum G_n(m) = 7 + \sum mg_n$

$$= 7 + g_n\sum m = 7 + 48g_n = 7 + 48(1+3^n) = 55 + 48\times3^n.$$

"55" is the number of corners of their 48 sectors.

The number of hexagonal yods in the seven separate nth-order polygons ≡ $H_n = \sum h_n(m)$

$$= (1+3^{n-1}+3^n)\sum m = 48(1+3^{n-1}+3^n) = 48 + 64\times3^n.$$

The number of yods in the seven polygons ≡ $Y_n = \sum Y_n(m) = \sum[1 + m(3+3^{n+1})/2 = 7 + 24(3+3^{n+1})$

$$= 7 + 72(1+3^n) = 79 + 8\times3^{n+2}.$$

Seven enfolded polygons

<u>Corners</u>

The number of corners per sector of an (n−1)th-order polygon = $c_{n-1} = (1+3^{n-2})/2$. Hence, the number of corners in a (n−1)th-order triangle = $1 + 3c_{n-1} = 1 + 3(1+3^{n-2})/2 = (5+3^{n-1})/2$. The number of corners of triangles in the seven separate, nth-order polygons = $\sum C_n(m)$

$$= 7 + ½(1+3^{n−1})∑m = 7 + 24(1+3^{n−1}) = \mathbf{31} + 8×3^n.$$

When the polygons are enfolded, a corner of the pentagon coincides with the centre of the decagon, a corner of the triangle coincides with the centre of the hexagon and the nth-order triangle replaces a sector (an (n−1)th-order triangle) of the nth-order hexagon. Also, single sides of five other polygons coincide with the root edge, so that the (5×2=10) corners that these sides join disappear. The number of corners of triangles in the seven enfolded nth-order polygons ≡ C_n = **31** + $8×3^n – 1 – 10 − (5+3^{n−1})/2$

$$= (35+47×3^{n−1})/2.$$

The number of corners in the (7+7) enfolded polygons = $2C_n − 2 = 33 + 47×3^{n−1}$.

Sides

The number of sides of triangles in an (n−1)th-order polygon = $ms_{n−1} = m(1+3^{n−1})/2$. Hence, the number of sides in an (n−1)th-order triangle = $3(1+3^{n−1})/2$. The number of sides in the seven separate nth-order polygons = $∑S_n(m) = ∑ms_n = ½(1+3^n)∑m = 24(1+3^n)$. The number of sides in the seven enfolded polygons ≡ $S_n = 24(1+3^n) – 5 − 3(1+3^{n−1})/2 = (35+47×3^n)/2$. The number of sides in the (7+7) enfolded polygons = $2S_n − 1 = 34 + 47×3^n$.

Triangles

The number of triangles in an (n−1)th-order polygon = $T_{n−1}(m) = 3^{n−2}m$. Hence, the number of triangles in an (n−1)th-order triangle = $3^{n−1}$. The number of triangles in the seven separate polygons = $∑T_n(m) = ∑3^{n−1}m$ = **48**$×3^{n−1} = 16×3^n$. The number of triangles in the seven enfolded polygons ≡ $T_n = 16×3^n − 3^{n−1} = 47×3^{n−1}$. The number of triangles in the (7+7) enfolded polygons = $2T_n = 94×3^{n−1}$.

Geometrical elements

The number of geometrical elements in the seven enfolded polygons ≡ $G_n = C_n + S_n + T_n = 35 + 47×3^n$. The number of geometrical elements in the (7+7) enfolded polygons = $2G_n − 3$ = **67** + $94×3^n$, where **67** is the number value of Binah. As the topmost corner of each hexagon coincides with the bottom of a hexagon enfolded in the next higher Tree of Life, the number of geometrical elements that are *intrinsic* to the inner form of each Tree = **65** + $94×3^n$, where **65** is the number value of ADONAI, the Godname of Malkuth.

Yods

The number of hexagonal yods in the seven enfolded polygons ≡ $H_n = 2S_n + T_n$

$$= 35 + 47×3^n + 47×3^{n−1} = 35 + 188×3^{n−1}.$$

The number of hexagonal yods in the (7+7) enfolded polygons = $2H_n − 2 = 68 + 376×3^{n−1}$.

The number of yods in the seven enfolded polygons ≡ $Y_n = C_n + H_n$

$$= (35 + 47×3^{n−1})/2 + 35 + 188×3^{n−1} = ½(105 + 47×3^{n+1}).$$

The number of yods in the (7+7) enfolded polygons = $2Y_n − 4$ = **101** + $47×3^{n+1}$. Amazingly, **101** is the **26**th prime number and 47 is the **15**th prime number, where **26** is the number value of YAHWEH, the Godname of Chokmah, and **15** is the number of YAH, the shorter version of the Godname. It demonstrates how this well-known Name of God prescribes through its gematria number the yod population of the inner form of the Tree of Life.

The number of yods surrounding the 14 centres of the (7+7) enfolded polygons = **87** + $47×3^{n+1}$, where **87** is the number value of *Levanah*, the Mundane Chakra of Yesod.

Table 10 lists the geometrical and yod compositions of the 7 enfolded nth-order polygons and the (7+7) enfolded nth-order polygons:

Table 10. Geometrical & yod compositions of the inner form of the Tree of Life.

	7 enfolded polygons	**(7+7) enfolded polygons**
Number of corners (including centres of polygons)	$C_n = ½(35 + 47×3^{n−1})$	$C = 2C_n −2 = 33 + 47×3^{n−1}$
Number of sides	$S_n = ½(35 + 47×3^n)$	$S = 2S_n − 1 = 34 + 47×3^n$
Number of triangles	$T_n = 47×3^{n−1}$	$T = 2T_n = 94×3^{n−1}$
Number of geometrical elements	$G_n = C_n + S_n + T_n = 35 + 47×3^n$	$G = C + S + T = 2G_n − 3 =$ **67** $+ 94×3^n$
Number of hexagonal yods	$H_n = 35 + 188×3^{n−1}$	$H = 2H_n − 2 = 68 + 376×3^{n−1}$
Number of yods	$Y_n = ½(105 + 47×3^{n+1})$	$Y = 2Y_n − 4 =$ **101** $+ 47×3^{n+1}$

Notice that the number **101** is the **26**th prime number and that the number 47, which is the number of

sectors of the seven enfolded polygons, is the **15**th prime number. The number **26** of YAHWEH, the complete Godname of Chokmah, and the number **15** of YAH, the shorter form of this Divine Name, arithmetically determine the yod population of the inner Tree of Life, whatever the order of its 14 polygons. Notice also that the number **67** of Binah appears in the formula for the geometrical population of the inner Tree of Life. It is the number of yods below Binah of the 1-tree when its 19 triangles are 1st-order triangles [1].

Let us call the seven enfolded nth-order polygons P_n. Table 10 indicates that the number of corners in $P_n \equiv C_n = ½(35 + 47×3^{n-1})$. Therefore, the number of corners in $P_{n+1} = C_{n+1} = ½(35 + 47×3^n)$. But, according to Table 10, this is the number of sides in P_n. We find that *the number of corners in a given P_n is equal to the number of sides in P_{n-1}, whilst its number of sides is equal to the number of corners in P_{n+1}.* The number of yods in P_n outside the shared root edge = $½(105 + 47×3^{n+1}) - 4 = ½($**97**$ + 47×3^{n+1})$. The number of *Haniel*, the Archangel of Netzach, is **97**.

The number of corners & triangles in $P_n = ½(35 + 47×3^n)$. According to Table 10, this is equal to the number of sides (S_n). This means that *the number (G_n) of corners, sides & triangles in P_n is twice its number of sides:*

$$G_n = 2S_n.$$

This property has its counterpart in a polygon of any order, namely, the number of corners & triangles surrounding its centre is twice the number of their sides [2]. For the (7+7) enfolded polygons, the number of corners & triangles = $33 + 47×3^n$, which is equal to the number of sides *outside* the root edge, whatever the order of polygon. These remarkable properties are clearly displayed in Table 11, which lists the geometrical and yod composition of the inner Tree of Life for n = 1, 2, 3 & 4:

Table 11

	7 enfolded polygons				**(7+7) enfolded polygons**			
	n = 1	**n = 2**	**n = 3**	**n = 4**	**n = 1**	**n = 2**	**n = 3**	**n = 4**
Number of corners (including centres of polygons)	41	88	229	652	**80**	174	456	1302
Number of sides	88	229	652	1921	175	457	1303	3841
Number of triangles	47	141	423	1269	94	**282**	846	2538
Number of geometrical elements	176	458	1304	3842	349	913	2605	7681
Number of hexagonal yods	223	599	1727	5111	444	1196	3452	10220
Number of yods	264	687	1956	5763	524	1370	3908	11522

In the case of the seven enfolded polygons:

for n = 1, 41 + 47 = 88; for n = 2, 88 + 141 = 229; for n = 3, 229 + 423 = 652; for n = 4, 652 + 1269 = 1921.

In the case of the (7+7) enfolded polygons:

for n = 1, **80** + 94 = 175 − 1; for n = 2, 174 + **282** = 457 − 1; for n = 3, 456 + 846 = 1303 − 1; for n = 4, 1302 + 2538 = 3841 − 1.

The number of yods in P_n with C_n corners and S_n sides of T_n triangles $\equiv Y_n = C_n + 2S_n + T_n$. As, according to Table 10,

$$C_n + T_n = S_n,$$

$$Y_n = 3S_n.$$

The number of corners, sides & triangles in $P_n \equiv G_n = C_n + S_n + T_n = 2S_n$. Therefore,

$$Y_n/G_n = 3/2.$$

The numbers in Table 11 confirm this for n = 1, 2, 3 & 4:

$\mathbf{Y_n/G_n = 3/2}$

264/176 = 687/458 = 1956/1304 = 5763/3842 = 3/2.

$\mathbf{Y_n = 3S_n}$

264 = 3×88, 687 = 3×229, 1956 = 3×652, 5763 = 3×1921.

$\mathbf{G_n = 2S_n}$

176 = 2×88, 458 = 2×229, 1304 = 2×652, 3842 = 2×1921.

What is familiar to musicians as the tone ratio 3/2 of the perfect fifth (namely, the arithmetic mean of the

tone ratios of the first and last notes of an octave — see here) is also the ratio of the yod and geometrical populations of the seven enfolded polygons, *whatever their order*. This is a remarkable property. The 0th-order triangle and the tetractys that corresponds to it have an analogous property, for nine yods line its boundary, which comprises three corners and three sides, i.e., six geometrical elements, and 9/6 = 3/2. Notice that for the root edge, the ratio of the number of yods (4) to the number of geometrical elements (3) is 4/3, which is the tone ratio of the perfect fourth! As the number of yods surrounding the centre of an nth-order N-gon = $y_n^N - 1 = (3/2)(3^n+1)N$ and the number of geometrical elements surrounding its centre = $g_n^N - 1 = (3^n+1)N$, the ratio of these two numbers for any nth-order N-gon is 3/2, whatever the values of n and N. In view of this property for individual polygons, it is not remarkable that the ratio of the sums of these two quantities for all seven separate, nth-order polygons is also 3/2. Indeed, it is true for any set of separate polygons, not just for those that make up the inner Tree of Life. What is non-trivial is that this same ratio applies to the populations of all yods and geometrical elements in the seven regular polygons when they are enfolded. **31** yods disappear when the separate polygons become enfolded, so that the yod population of 295 is reduced to 264. Twenty-three geometrical elements disappear, so that the 199 geometrical elements become 176 elements. The 288 yods surrounding the centres of the seven separate polygons become the 257 yods that surround the centres of the seven enfolded polygons, whilst the 192 geometrical elements surrounding centres of the seven separate polygons become 169 geometrical elements surrounding their centres when enfolded. Despite this disappearance, the fraction 3/2 survives the enfoldment, this time as the ratio of the *total* populations of yods and geometrical elements instead of as the ratio of numbers of yods and elements surrounding centres of the seven separate polygons (notice that 288/192 = 3/2 but 257/169 ≠ 3/2, whereas 264/176 = 3/2)! This would not happen for *any* set of polygons. In fact, the property exists only for a set of enfolded polygons that includes either a pentagon & decagon or a triangle, pentagon, hexagon & decagon (as in the case of the inner Tree of Life) [3].

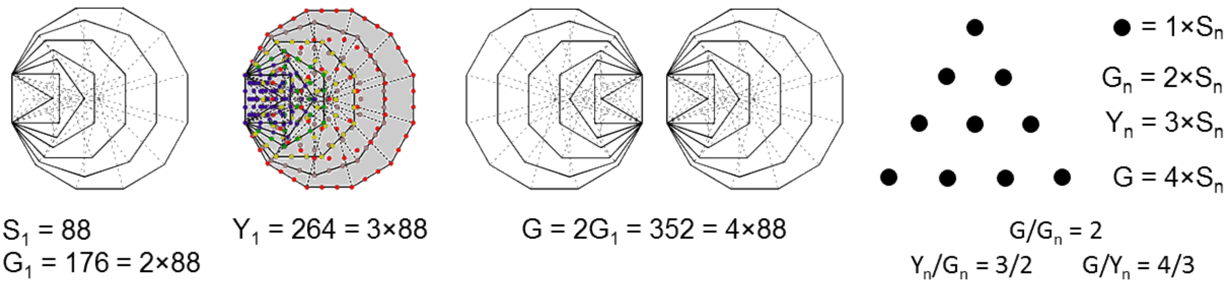
As $Y_n = 3S_n$, the number of yods in two separate sets of seven enfolded polygons ≡ $Y' = 2Y_n = 6S_n$ and their number of geometrical elements ≡ $G' = 2G_n = 4S_n$. Therefore,

$$Y'/G' = 3/2$$

and

$$G'/Y_n = 4/3.$$

This is the tone ratio of the perfect fourth in music. The ratio $Y'/Y_n = 2$ corresponds to the octave. These three ratios are the ratios generated by the tetractys in the historical context of Pythagoras' contribution to music theory that is well-known to students of music and science:



The tone ratios of the octave, perfect 5th & perfect 4th appear in the proportions of various populations of yods and geometrical elements in the two *separate* halves of the inner Tree of Life (the case of 1st-order polygons is chosen as an example).

What is the corresponding relation between the yod (Y) and geometrical (G) populations of the (7+7) *enfolded* polygons? As

$$Y = 2Y_n - 4$$

and

$$G = 2G_n - 3,$$

we find that

$$2Y = 3G + 1.$$

This compares with the relation

$$2Y_n = 3G_n$$

for the seven enfolded, nth-order, regular polygons making up half of the inner form of the Tree of Life. As $Y = 2Y_n - 4$, $G = 2G_n - 3$, $Y_n = 3S_n$ and $G_n = 2S_n$,

$$Y = 6S_n - 4$$

and

$$G = 4S_n - 3.$$

The total number of sides in the (7+7) enfolded polygons ≡ $S = 2S_n - 1 = 34 + 47\times3^n$. Therefore,

$$Y = 3S - 1$$

and

$$G = 2S - 1,$$

so that

$$Y/G = (3S-1)/(2S-1).$$

This may be simplified to:

$$Y/G = 3/2 + ½(2S-1)^{-1}.$$

As $n\rightarrow\infty$, $S\rightarrow\infty$ and $Y/G\rightarrow3/2$. The ratio of the yod & geometrical populations of the complete inner form of the Tree of Life is larger than the fraction 3/2 but *reduces as $n\rightarrow\infty$ from its maximum value of 524/349 when n = 1 to 3/2 as its asymptotic limit for an infinitely populated inner Tree of Life.* This amounts only to a decrease of about 0.095%. Even when n = 1, the ratio is only about 0.1% above 3/2. But, for the seven enfolded polygons, the ratio remains exactly 3/2 for *all* values of n.

In terms of their number (C) of corners of their T triangles, G = C + S + T. Therefore,

$$C + S + T = 2S - 1,$$

so that

$$S = C + T + 1.$$

If we define S′ as the number of sides outside the root edge of the (7+7) enfolded, nth-order polygons, then S′ = S − 1. This means that

$$S' = C + T.$$

We find that *the number of external sides is equal to the number of their 0th-order triangles and their corners*. This is a remarkable, geometrical property of the inner form of the Tree of Life. Therefore, as G = C + S + T,

$$G = S + S'.$$

The number of geometrical elements in the inner Tree of Life is the sum of the number of sides of all its triangles and the number outside the root edge shared by the (7+7) enfolded polygons. The figures in Table 10 illustrate this:

349 = 175 + 174.

913 = 457 + 456.

2605 = 1303 + 1302.

7681 = 3841 + 3840.

"Intrinsic yods" are discussed in many pages of this website. They are defined as all the yods in the inner form of the Tree of Life except the topmost corners of the two hexagons, which are shared with the lowest corners of the two hexagons belonging to the inner form of the next higher, overlapping Tree of Life. Intrinsic yods are those unshared yods that *solely* make up the inner form of each successive Tree. "Intrinsic geometrical elements" are, similarly, discussed many times. They are all those elements, apart from the topmost, shared corners of the pair of hexagons, that uniquely shape the inner form of each overlapping Tree because they are unshared with polygons enfolded in the next higher Tree of Life. Two other sets of yods and geometrical elements may be defined:

1. the yods or geometrical elements *outside* the root edge that create the form of the inner Tree of Life;
2. the intrinsic yods or geometrical elements *outside* the root edge. Table 12 below lists the relations between the populations of these types of yods and geometrical elements in the seven enfolded

polygons and the (7+7) enfolded polygons (n.b. the subscript n is dropped from symbols because the relation: $2Y_n = 3G_n$ holds whatever the value of n, although both populations are, of course, functions of n. But, for the purpose of Table 12, the change of symbols: $Y_n \rightarrow y$ & $G_n \rightarrow g$ will be made, with Y & G still referring to the (7+7) enfolded polygons).

Table 12

	7 enfolded polygons	**(7+7) enfolded polygons**
Number of yods Number of geometrical elements Relation	y g $2y = 3g$	$Y = 2y - 4$ $G = 2g - 3$ $2Y = 3G + 1$
Number of yods outside root edge Number of geometrical elements outside root edge Relation	$y' \equiv y - 4$ $g' \equiv g - 3$ $2y' = 3g' + 1$	$Y' \equiv Y - 4$ $G' \equiv G - 3$ $2Y' = 3G' + 2$
Number of intrinsic yods Number of intrinsic geometrical elements Relation:	$y_0 \equiv y - 1$ $g_0 \equiv g - 1$ $2y_0 = 3g_0 + 1$	$Y_0 \equiv Y - 2$ $G_0 \equiv G - 2$ $2Y_0 = 3G_0 + 3$
Number of intrinsic yods outside root edge Number of intrinsic geometrical elements outside root edge Relation:	$\hat{y} \equiv y_0 - 4$ $\hat{g} \equiv g_0 - 3$ $2\hat{y} = 3\hat{g} + 2$	$\hat{Y} \equiv Y_0 - 4$ $\hat{G} \equiv G_0 - 3$ $2\hat{Y} = 3\hat{G} + 4$

Notice in Table 12 that the relations between types of populations contain only the Pythagorean integers 1, 2, 3 & 4 expressed by the four rows of the tetractys. In particular, these integers are the constant that appears in the relations for the complete inner form of the Tree of Life. Their presence is indicating that these four classes of yods and geometrical elements are special. Much of the analysis in the pages of

this website is devoted to them because they generate numbers and patterns that manifest in the group mathematics of $E_8 \times E_8$ and in the remote-viewing accounts of superstrings given by Annie Besant & C.W. Leadbeater (see Occult Chemistry), as well as in the seven musical diatonic scales and in the correspondences between the inner form of the Tree of Life and other sacred geometries, as described in Correspondences and Wonders of correspondences.

The number of yods that line the S_n sides of the T_n triangles in $P_n \equiv B_n = C_n + 2S_n$. As $G_n = 2S_n$,

$$B_n = C_n + G_n = ½(105 + 329 \times 3^{n-1}).$$

The number of boundary yods in the (7+7) enfolded nth-order polygons = $2B_n - 4$ = **101** + $329 \times 3^{n-1}$. Table 13 lists the boundary yods for n = 1, 2, 3 & 4:

Table 13

	7 enfolded polygons				**(7+7) enfolded polygons**			
	n = 1	**n = 2**	**n = 3**	**n = 4**	**n = 1**	**n = 2**	**n = 3**	**n = 4**
Number of boundary yods	217	546	1533	4494	430	1088	3062	8984

In particular, the number of boundary yods shaping the seven enfolded, 4th-order polygons outside their root edge = 4494 − 4 = 4490 = 449×10, where 449 is the **87**th prime number and **87** is the number value of *Levanah*, the Mundane Chakra of Yesod.

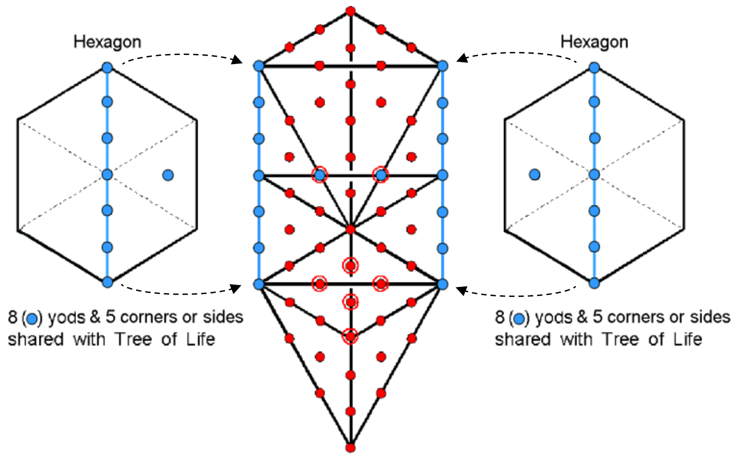
As the top corners of both hexagons in the (7+7) polygons enfolded in any given Tree of Life coincide with the lowest corners of the hexagons enfolded in the next higher, overlapping Tree of Life, there are (430−2=**428**) yods lining these (7+7) 1st-order polygons that are intrinsic to them alone. This is the number value of *Chasmalim*, the Order of Angels assigned to Chesed. Table 13 indicates that 217 yods line the seven enfolded polygons. (217−1=**216**) *intrinsic* yods line the 88 sides of the 47 tetractyses making up each set of enfolded 1st-order polygons with **36** corners, where **216** is the number value of Geburah and **36** is the number value of ELOHA, its Godname.

3. Geometrical & yod populations of the combined outer/inner N Trees and N-tree

When the outer form of the Tree of Life combines with its inner form, the number of geometrical elements or yods in the combination is not the sum of the respective numbers of both forms. This is because certain elements and yods coincide when they combine. They are indicated in the diagram below as blue yods

and lines. Remembering that the plane occupied by the (7+7) enfolded polygons is that formed by the outer Pillars of the Tree of Life, each nth-order hexagon has six yods (two of which are corners) and two vertical sides of sectors that are shared with the Pillars of Judgement and Mercy and the horizontal Path joining Chesed and Geburah. Each enfolded nth-order triangle that occupies one of the sectors of each hexagon has one corner and one centre that coincide with yods in the outer Tree. It means that 16 yods (including six corners of triangles) and four sides are shared by the outer and inner forms of the Tree of Life, so that the combination needs 16 yods and 10 geometrical elements (six corners & four sides) to be subtracted from the sums of the respective, separate populations.

For mathematical consistency, when the inner form of the Tree of Life is regarded as composed of 14 nth-order polygons, the triangles making up its outer form must be regarded only as nth-order. It would be inconsistent to combine 1st-order polygons with 0th-order triangles in the outer Tree as shown in the diagram, which is intended merely to display what yods and what geometrical elements become shared during the combination of the outer and inner forms — whatever the same order of their polygons and triangles. This consistency must be maintained when the outer and inner forms of N overlapping Trees or the N-tree are considered. What also must be considered is the fact that the two blue, hexagonal yods on the Chesed-Geburah Path remain hexagonal yods, whatever the order of the triangles in the outer Tree of Life, whereas the blue yod (with which they coincide) at the centre of each triangle in the (7+7) polygons remains a *corner* of a 0th-order triangle, whatever the order of these polygons. It means that there are two unique yods in the combined outer & inner forms of the Tree of Life (and for every overlapping Tree) that are hexagonal yods with respect to its outer form but which are corners with respect to its inner form. Their dual status must be taken into account in calculating the hexagonal yod population of the combined Trees. Finally, the topmost corners of the two hexagons enfolded in a given Tree of Life coincide with the lowest corners of the two hexagons enfolded in the next higher Tree. This means that the number of yods shared by the outer and inner forms of N overlapping Trees of Life is 14N + 2, where "2" denotes the topmost corners of the two hexagons enfolded in the Nth Tree. It also means that the number of geometrical elements shared by both forms is 8N + 2. Both these quantities need to be subtracted from the sum of the separate populations of yods and geometrical elements to avoid double-counting when the two forms are combined.



Using Tables 4, 7 & 10, the calculated yod and geometrical compositions of the combined outer and inner forms of N Trees/N-tree when the former have nth-order triangles and the latter have nth-order polygons are shown in Table 14:

Table 14. Numbers of corners, geometrical elements & yods in combined forms of N Trees/N-tree.

	Combined outer & inner forms	
	N Trees of Life	**N-tree**
Number of corners	$2 + 27N + (\mathbf{65}N+6)\times 3^{n-1}$	$\frac{1}{2}[3 + 54N + (130N+\mathbf{21})\times 3^{n-1}]$
Number of geometrical elements	$2 + 55N + 2(\mathbf{65}N+6)\times 3^{n}$	$2 + 55N + (130N+\mathbf{21})\times 3^{n}$
Number of hexagonal yods	$54N + 8(\mathbf{65}N+6)\times 3^{n-1}$	$54N - 3 + (520N+84)\times 3^{n-1}$
Number of yods	$2 + 81N + (\mathbf{65}N+6)\times 3^{n+1}$	$\frac{1}{2}[162N - 3 + (130N+\mathbf{21})\times 3^{n+1}]$

Notice that ADONAI, the Godname of Malkuth with number value **65**, prescribes the yod population of the combined outer & inner forms of N Trees of Life, as well as their number of corners and geometrical elements, the number 130 being the **65**th even integer. *Here is remarkable evidence that the ancient Hebrew Divine Names determine through their gematria number values the very mathematical nature of the cosmic blueprint called the "Tree of Life."*

The increase in yods from N Trees to (N+1) Trees = 81 + **65**$\times 3^{n+1}$. For n = 1, the increase is 666. This is

the **36**th triangular number, showing how ELOHA, the Godname of Geburah with number value **36**, measures how many yods are needed to build the outer and inner forms of *successive* Trees of Life. The *average* number of hexagonal yods that surround the axes of the five Platonic solids in their faces and interiors is 666 (see bottom of page here).

Table 15 lists the populations of geometrical elements & yods in the Tree of Life and the 1-tree for 1st-, 2nd-, 3rd- & 4th-order triangles/polygons:

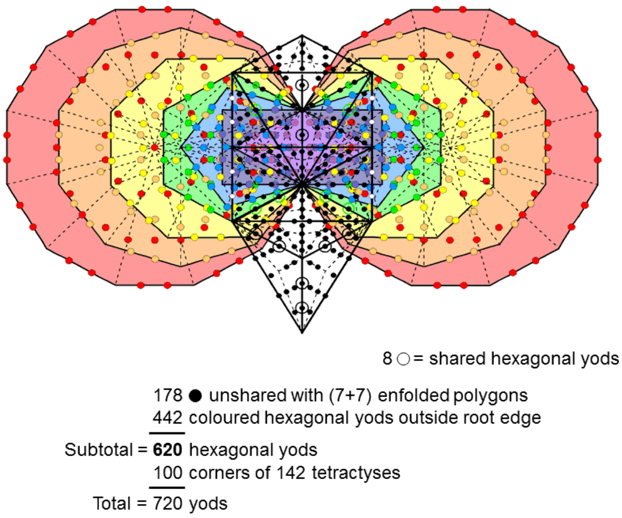
Table 15

	Combined outer & inner forms							
	Tree of Life				1-tree			
	n = 1	n = 2	n = 3	n = 4	n = 1	n = 2	n = 3	n = 4
Number of corners	100	242	668	1946	104	255	708	2067
Number of geometrical elements	483	1335	3891	11559	510	1416	4134	12288
Number of hexagonal yods	622	1758	5166	15390	655	1863	5487	16359
Number of yods	722	2000	5834	17336	759	2118	6195	18426

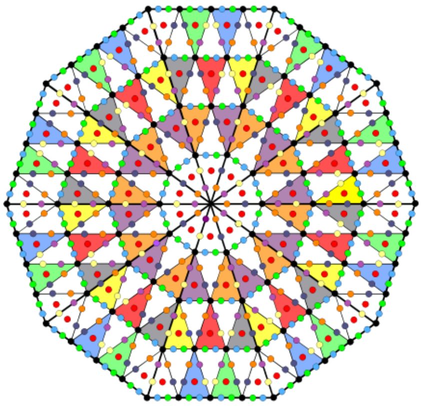
The combined outer & inner forms of the Tree of Life with 1st-order triangles and polygons comprise 483 geometrical elements. Of these, three make up the root edge, so that 480 elements are outside it. This number is a characteristic parameter of holistic systems, e.g., the 480 hexagonal yods in the (7+7) separate polygons of the inner Tree of Life (see here) and the 480 hexagonal yods in the faces of the first four Platonic solids constructed from tetractyses (see here). In the case of the $E_8\times E_8$ heterotic superstring, it is the number of roots of $E_8\times E_8$ (see under heading "Superstring gauge symmetry group" here).

As the 1-tree shares 10 geometrical elements with its inner form (namely, the three corners and two sides lining the Pillars of Mercy & Judgement), the combined forms comprise 510 geometrical elements, of which 500 (=**50**×10) are unshared by either form. This shows how ELOHIM, the Godname of Binah with number value **50**, prescribes the combined, outer & inner forms of the 1-tree.

Of the 622 hexagonal yods in the combined forms of the Tree of Life made up of 1st-order triangles and polygons, two belong to the root edge. **620** hexagonal yods are outside it. This number is the number value of Kether (see here). Of the 722 yods, two hexagonal lie on the root edge, leaving 720 yods that comprise 100 corners and **620** hexagonal yods. This 100:**620** division of the number 720 is identical to that displayed by the decagon when its sectors are 2nd-order tetractyses:



The 720 yods in the combined Trees of Life other than the two hexagonal yods in the root edge comprise 100 corners and **620** hexagonal yods.



The 720 yods surrounding the centre of a decagon with 2nd-order tetractyses as sectors comprise 100 corners of 1st-order tetractyses and **620** hexagonal yods.

Its 100 tetractyses have 720 yods surrounding its centre. They comprise 100 corners and **620** hexagonal yods. This embodiment of the number **620** of Kether (the first of the 10 Sephiroth) by two 10-fold representations of God, one Kabbalistic, the other of a Pythagorean character, demonstrates the

mathematical parallels between different sacred geometries when they are *truly* such. See here for further analysis of how the combined Trees of Life embody this number, as well as other Kabbalistic numbers. Other examples of the embodiment of the number 720 by sacred geometries are:

- the five Platonic solids constructed from tetractyses, which have 720 hexagonal yods in their faces (see here);
- the 720 yods surrounding the centres of the seven separate, 2nd-order polygons of the inner Tree of Life (see Fig. 7 here);
- the 720 corners of triangles surrounding the axis of the disdyakis triacontahedron that make up its Type A triangular faces and internal triangles formed by its edges and face sector sides (see here);
- the 720 sides & triangles surrounding the axis in each half of the first four Platonic solids with their faces divided into sectors when their interior triangles formed from edges and sector sides are Type A (see here);
- the 720 edges of the 600-cell (see below Table 1a here).

References

1. See here.

2. Proof: an nth-order N-gon is one whose N sectors are (n−1)th-order triangles. Using Table 9, the number of corners surrounding the centre of an nth-order N-gon ≡ $c_n^N = \frac{1}{2}(3^{n-1}+1)N$,

the number of sides of triangles in an nth-order N-gon ≡ $s_n^N = \frac{1}{2}(3^n+1)N$,

and the number of triangles in an nth-order N-gon ≡ $t_n^N = 3^{n-1}N$.

Therefore,

$$c_n^N + t_n^N = \tfrac{1}{2}(3^{n-1}+1)N + 3^{n-1}N = \tfrac{1}{2}(3^n+1)N = s_n^N$$

and

$$c_n^N + s_n^N + t_n^N = 2s_n^N.$$

3. Proof: consider a set (P) of m different, separate nth-order polygons with N corners. According to Table 9, the number of yods in an nth-order r-gon = $y_n^r = (3/2)(3^n+1)r + 1$.

Number of geometrical elements = $g_n^r = (3^n+1)r + 1$.

Number of yods in P ≡ $Y = \sum y_n^r = (3/2)(3^n+1)N + m$.

Number of geometrical elements in P ≡ $G = \sum g_n^r = (3^n+1)N + m$.

The ratio (Y−m)/(G−m) of the numbers of yods and geometrical elements surrounding the centres of the m separate nth-order polygons is always 3/2. Suppose that p yods and q geometrical elements outside the sides that merge into the root edge vanish when the m polygons become enfolded. These are not the total numbers that vanish because the 4(m−1) yods and 3(m−1) geometrical elements making up the (m−1) sides that merge into the root edge of the enfolded polygons disappear as well.

Number of yods in m enfolded nth-order polygons ≡ $Y_n(m) = Y - p - 4(m-1) = (3/2)(3^n+1)N + 4 - 3m - p$.

Number of geometrical elements in m enfolded polygons ≡ $G_n(m) = G - q - 3(m-1)$

$$= (3n+1)N + 3 - 2m - q.$$

Therefore, $Y_n(m)/G_n(m) = 3/2$ if $3q - 2p = 1$. Yods and geometrical elements can disappear only for two pairs of polygons: triangle-hexagon (let us call it "pair 1") and pentagon-decagon ("pair 2"). This is because the triangle takes the place of one sector of the hexagon, sharing six yods and four geometrical elements, and a corner of the pentagon coincides with the centre of the decagon, so that one yod and one geometrical element disappears when they become enfolded. Either p = 1 (P includes only pair 2), 6 (P includes only pair 1) or 7 (P includes pairs 1 & 2) and q = 1 (P includes pair 2), 4 (P includes pair 1) or 5 (P includes pairs 1 & 2). The relation between p and q:

$$3q - 2p = 1$$

derived above does not hold for p = 0 = q, so that P must include at least one of the two pairs of polygons that share yods and geometrical elements. Therefore, $Y_n(m)/g_n(m) = 3/2$ only if P includes:

- just pair 1. But then p = 6 and q = 4, which does not satisfy the relation. This means that P must include at least pair 2;
- just pair 2. Then p = 1 = q, which satisfies the relation.

- pairs 1 & 2. Then p = 7 and q = 5, which obeys the relation.

It is concluded that $Y_n(m)/G_n(m) = 3/2$ only if P includes either pair 2 (but not pair 1) or both pairs, i.e., m≥2. In the case of the inner Tree of Life, the 14 enfolded polygons have 70 corners that correspond to the 70 yods in the outer Tree of Life constructed from tetractyses. This means that either set of seven enfolded polygons has **36** corners. If we insist that the 2m enfolded polygons must have 70 corners to match the 70 yods, then the m enfolded polygons must have **36** corners. Separate, they have N corners; when enfolded, they have [N − 2(m−1)] corners because (m−1) sides disappear into the root edge. Therefore,

$$\mathbf{36} = N - 2(m-1) = N - 2m + 2,$$

so that

$$N = 34 + 2m.$$

As m≥2, N≥38. P cannot be just pair 2, because they have only **15** corners. Nor can it be just pair 1 & pair 2, because they have only 24 corners. This means that m>2 and N>38. As the polygons must include the pentagon and the decagon, the remaining (m−2) separate polygons must have more than (38−**15**=23) corners. If they include both pairs of polygons, m>4 and N>(34+8=42), so that the remaining (m−4) separate polygons must have more than (42−24=18) corners.

The 1-tree has **80** yods (see here). They are matched by the **80** corners of the 94 sectors of the (7+7) enfolded polygons. If we insist that the 2m enfolded polygons must have sectors with **80** corners, then each set of m enfolded polygons must have sectors with 41 corners. Suppose that the centres of A polygons coincide with their corners when enfolded. As P must include at least pair 2, A = 1 if P includes only pair 2 (m>2) and A = 2 if it includes both pairs (m>4). Separate, the sectors of the polygons have (N+m) corners; when enfolded, they have [N − 2(m−1) + m − A] corners. Therefore,

$$41 = N + 2 - m - A,$$

so that

$$N = 39 + m + A.$$

But

$$\begin{aligned} m + A &> 3 \text{ (P includes only pair 2)} \\ &> 6 \text{ (P includes pairs 1 \& 2).} \end{aligned}$$

This means that either N>42, so that the remaining (m−2) polygons must have more than (42−**15**=27) corners, or N>45, so that the remaining (m−4) polygons must have more than (45−24=**21**) corners. If we insist that the 2m enfolded polygons have 70 corners and that their sectors have **80** corners, then P must include the pentagon and the decagon and the remaining (m−2) polygons must have more than 27 corners, or else it must include the triangle, pentagon, hexagon & decagon and the remaining (m−4) polygons must have more than **21** corners. In the case of the inner Tree of Life, the three remaining polygons (square, octagon & dodecagon) have 24 corners and so, as expected, the seven enfolded polygons display the property: $Y_n(m)/G_n(m) = 3/2$.